Bank Competition, Bank Runs and Opacity*

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Abstract

We examine the competition among and the opacity of banks subject to rollover risk. Banks imperfectly compete for uninsured deposits and choose the opacity of their risky investment. In a static setup, higher bank competition increases the deposit rate, which increases withdrawal incentives due to strategic complementarity and thus raises bank fragility. In a dynamic setup with entry, a theory of bank opacity arises. Opacity trades off a static cost of larger withdrawals and costly liquidation of investment with a dynamic benefit of deterring entry and reducing future competition. We use our framework to evaluate the regulation of competition or transparency. We find that greater competition increases deposit rates, fragility, and transparency, while minimum transparency regulation increases both current and future fragility and future competition.

Keywords: Competition, entry, fragility, global games, rollover risk, opacity.

JEL classifications: G01, G21, G28.

1 Introduction

The recent financial crisis demonstrated the relevance of bank failures for the economy and the need for understanding their causes. A key determinant of bank failure is the competitive intensity in the banking industry. The main argument put forward in the literature is that competition among banks affects the risk-taking decisions of banks or firms and thus bank failure. However, this strand of the literature has overlooked one key mechanism emphasized in recent empirical studies as a key determinant of bank failures: the withdrawal decisions of investors—commonly known as bank runs.[1]

This paper offers a parsimonious model of imperfect competition among banks who choose their opacity and are subject to runs. This workhorse model allows to study the effect of changes in economic conditions (e.g., bank competition and secular changes in transparency) on bank failure, emphasizing changes in the incentives to withdraw as a novel channel.[2] A more competitive banking system results in higher deposit rates that increase the incentives to withdraw from the bank, leading to more fragile banks. Higher deposit rates increase the strategic complementarities of withdrawal decisions because a given withdrawal has a larger negative impact on the residual funds available and thus increase the withdrawal incentives of other investors.

The model allows us to examine bank opacity choices and their implications for financial stability and competitive intensity. We characterize these choices as well as the impact of regulatory interventions aimed at reducing the opacity in the banking sector. We uncover a novel trade-off for bank opacity. On the one hand,

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[1]For theoretical work, see Diamond and Dybvig (1983) for seminal work on bank runs and Goldstein and Pauzner (2005), Rochet and Vives (2004), and Morris and Shin (2000), among others, for applications of global-games methods to runs. For evidence of runs in the banking industry in the recent financial crisis, see Ivashina and Scharfstein (2010) and Ippolito et al. (2016), among others.

[2]The bank competition literature largely focused on the risk choices of banks or entrepreneurs and the resulting asset-side risk. We highlight debt withdrawals and the risk on a bank's liability side. To the best of our knowledge, we are the first banking paper to integrate a model of competition and fragility. Work that emphasizes competition and the liability side of banks include Matutes and Vives (1996), who study bank failure but focus on sunspots, and Carletti and Leonello (2019), who study credit market competition and fundamental runs. See also Vives (2010).

banks have an incentive to be transparent to reduce costly withdrawals suffered from imperfectly informed investors. On the other hand, banks have an incentive to be opaque to reduce the incentives of potential competitors to enter, which would reduce future bank profits via higher competition for funding. When banks are more opaque, entrants have worse information about underlying economic conditions, reducing their incentives to enter. By being more opaque, a bank has lower incentives to attract funding by setting high deposit rates, reducing both withdrawal incentives and fragility. This result on opacity arises only once endogenous runs and competitive intensity in the banking sector is taken into account and motivates our global-games approach (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005).

We start our analysis by presenting a static setup in which a fixed set of banks compete for uninsured funding from investors located in a circle (Salop, 1979). Banks offer a debt contract that can be withdrawn upon demand and invest the received funds in a risky technology.[3] Each investor deposits its endowment with a bank and delegates the withdrawal decisions to a fund manager who receives a noisy private signal about the investment return and decides whether to withdraw (Rochet and Vives, 2004; Vives, 2014). To serve withdrawals, banks liquidate investment at a cost. We assume that the investment return and private signals are uniformly distributed, so bank opacity has no direct effect on bank failure. This approach allows us to isolate the impact of opacity via the competitive intensity.

We characterize the equilibrium deposit rate and failure threshold and show that a more competitive banking system (defined by a larger number of banks) results in higher deposit rates. In turn, this increases the incentives to withdraw, so higher competition results in a more fragile banking system. This result is in line with the view that competition can erode bank stability (e.g., Keeley (1990)). However, the mechanism does not arise from a direct risk-taking choice by banks on their assets

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[3]Consistent with much evidence, unsecured bank debt is assumed to be demandable. Demandability arises endogenously with liquidity needs (Diamond and Dybvig, 1983), as a commitment device to overcome an agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001).

but from a more fragile liability structure. Thus, we offer a novel mechanism of how more competitive banking sectors can result in higher bank failure: an increase in the withdrawal incentives and, thus, the run probability. The resulting testable implication is that an (exogenous) increase in bank competition increases the probability of debt runs and, therefore, bank failure.

We also characterize bank opacity, whereby signals about investment returns are noisier for banks that choose to be more opaque.[4] Hence, it is harder for outsiders to precisely learn about the actual returns of the bank. In the static setup, banks are fully transparent to lower partial withdrawals when banks are profitable. Since partial withdrawals result in costly liquidation of investment, more transparency increases bank profits. This result provides a micro-foundation for the common assumption of vanishing signal noise in many global-games bank run models (see literature review).

Next, we study the effects of a more transparent banking system and show how it can increase bank fragility. Higher transparency could arise from exogenous technological changes that generate better information. More transparent banks face lower partial withdrawals and, thus, have higher profits. As a result, banks compete more intensively for funding and offer higher deposit rates that, for the reasons explained above, increase the incentives to withdraw from the bank. Critically, this result only arises once the competitive intensity of the banking system is taken into account because our distributional assumptions imply no direct effect of opacity on fragility.

To illustrate the relevance of our model, we show how a safer banking industry results from circumstances with exogenously lower bank profits, for example because of the mentioned increase in transparency or an increase in a non-pecuniary operating cost (effort). A reduction in bank profits lowers deposit rates that, in turn, reduce the risk of bank failure. Critically, these results are opposed to those when risk originates on the asset side of the bank balance sheet (Keeley, 1990; Boyd and Nicolo, 2005).

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[4]Transparency has already been defined as the private signal precision in a global game of regime change in Heinemann and Illing (2002); Bannier and Heinemann (2005); Moreno and Takalo (2016).

In sum, the impact of bank competition on bank failure depends on whether risks emerge on the asset or liability side of the balance sheet of the bank.

Turning to a dynamic model with two periods, we examine the entry and opacity decisions of banks and their effects on the competitive structure and fragility of the banking sector. Banks choose both their deposit rates and opacity levels at the beginning of each period. Investment returns have some persistence, whereby the expected return in period 2 equals the realized return in period 1. The novel ingredient is that a new bank chooses in the first period whether to enter the market and operate in period 2. In this case, the entrant has to pay a fixed cost in period 1 before the investment returns are realized. The entrant, in parallel to fund managers, receives a noisy private signal about the return. Investors live for one period but banks that do not fail in period 1 also operate in period 2. To keep the benchmark competitive structure of the banking system constant across periods, we assume that if a bank fails in period 1 it is replaced by a new bank in period 2. Thus, the only change in the competitive structure arises from entry, and not from bank defaults.[5]

We solve of the dynamic model by backwards induction. The equilibrium in period 2 is the same as in the static model. We show that the equilibrium in period 1 differs from the static equilibrium. The reasons is that banks take into account how their choice of the deposit rate and the opacity level in the first period affect both (i) the chances of obtaining future profits (in line with the traditional charter value hypothesis) and (ii) the expected competition in the second period via the entry incentives (a novel mechanism). We characterize the optimal deposit rate and opacity level of banks in the dynamic setup and show that banks offer lower deposit rates in period 1 than in period 2. This result is due to the fact that banks internalize the loss of charter value upon default. This incentive leads banks to be less aggressive in the deposit market, resulting in lower bank fragility in the first period.

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[5]This assumption may reflect the fact that in case of failure the bank is acquired by an external bank and this acquisition may even be fostered by regulators.

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The dynamic model offers a theory of bank opacity. We show that banks choose an interior level of opacity in period 1. Each bank internalizes that higher opacity makes the signal about the investment returns noisier, which has has two opposing effects in period 1. It leads to a higher probability of fund managers withdrawing even in situations in which the underlying return is high, which makes banks have to partially liquidate some of its assets and, as previously explained, reduce their profits. However, by making the signal about their realized returns noisier, the potential entrant may also have a less precise signal about the future profitability upon entry. This noisier signal reduces the expected profits of the entrant and can deter entry and, thus, increase the incumbents expected charter value via lower future competition.

Using the dynamic model, we evaluate policies aimed at guaranteeing a minimum level of transparency (maximum opacity). Regulatory policies that increase transparency include changes in accounting rules, pillar 3 of Basel bank regulation, and the implementation of bank stress tests. We show that these policies have the consequence of increasing bank fragility in both periods. As in the static model, this policy increases per-unit expected bank profits and thus increases bank fragility in the first period. In the dynamic model, higher future expected profits also increase the incentives to enter. The higher competitive intensity of the banking sector upon entry results in higher fragility. Moreover, transparency policies have different short-term and long-term effects on bank profits, where an increase in transparency results in higher profits in period 1 but lower profits in period 2.

Next, we analyze the effects of greater bank competition and lower entry cost on bank deposit rates, opacity levels, and bank fragility. In the dynamic model, a more competitive banking sector results in higher deposit rates, more transparency, and higher fragility. Greater bank competition induces fiercer competition among banks for funding and lower charter value, where both effects combine to increase bank deposit rates. As a result, the threshold of the investment return below which banks fail increases. Since the entrant receives a signal from the nearest bank, the

value of using bank opacity to deter entry decreases in bank competition, resulting in a more transparent banking system. Similarly, a lower fixed cost of entry increases the incentives to enter and, therefore, leads to a more fragile and more transparent banking sector that offers higher deposit rates. The testable implications about how greater bank competition or lower entry costs reduce bank opacity are consistent with evidence presented by Jiang et al. (2016), who show that bank deregulation (a proxy for higher bank competition and lower entry barriers) leads to higher bank opacity.

This paper is related to several strands of the literature. First, a long-standing literature studies how bank failure is determined by the competitive intensity in the banking system. The traditional approach (e.g., Keeley (1990); Hellmann et al. (2000); Allen and Gale (2004)) highlights how bank incentives to take risk increases in bank competition in the presence of moral hazard, resulting in a higher probability of bank failure. A recent literature (Boyd and Nicolo, 2005) highlights that this result might not be true when, following Stiglitz and Weiss (1981), the risk choice is the result of a moral hazard problem of the entrepreneur, so higher competition results in lower loan rates and safer entrepreneurs.[6] Our contribution to this literature is to provide a novel mechanism based on withdrawal decisions in funding markets. Higher competition increases deposit rates and, therefore, increases the incentives to withdraw, which makes the bank more fragile and more prone to be run upon. The new testable implication is that higher bank competition (i) increases deposit rates and (ii) increases withdrawals of uninsured and unsecured funding.

Second, the paper is related to the literature on runs on financial intermediaries (Bryant, 1980; Diamond and Dybvig, 1983). We are closely related to the global games approach that uniquely pins down the run probability (Goldstein and Pauzner, 2005; Rochet and Vives, 2004) and, therefore, allows us to examine the impact of competition and opacity on bank fragility.[7] A contribution of our paper is to

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[6]Martinez-Miera and Repullo (2010) show that a non-monotonic relation between bank competition and stability arises when loan defaults are imperfectly correlated.

[7]Recent work on runs in a global games setup includes Vives (2014); Ahnert et al. (2019); Allen

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microfound the common assumption of vanishing private noise in bank run models. We show in the static model that profit-maximizing banks choose to be as transparent as possible.[8] Finally, we share endogenous deposit rates and runs with Goldstein and Pauzner (2005). By relaxing the assumption of perfect competition, we can examine the impact of competition in the banking sector on the fragility and opacity of banks.

Third, a strand of literature analyses bank opacity. Moreno and Takalo (2016) identify a non-monotonic welfare impact of transparency when a bank is subject to rollover risk. Their tradeoff is between efficient liquidation and ex-ante risk-taking of banks. Since liquidation is always inefficient in our model without risk-taking, our emphasis is on endogenous competition and we offer a novel theory of bank opacity, whereby a bank balances the static benefit of avoiding costly liquidation with the dynamic cost of causing entry of competitors. Jungherr (2018) models bank opacity about their portfolio composition and examines the role of public disclosure. For a survey on the benefits and costs of disclosure, see Goldstein and Sapra (2014).

2 Static model

We start by studying a model that combines bank runs in the tradition of Rochet and Vives (2004) with funding market competition in the tradition of Salop (1979).[9]

There are three dates $t = 0, 1, 2$, no discounting and universal risk neutrality. There are three types of agents: banks, investors, and fund managers. At date 0, each bank has access to the same risky investment technology with gross return R

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et al. (2018); Eisenbach (2017). Bouvard et al. (2015) model the disclosure about the health of banks subject to rollover risk. Ma and Freixas (2015) study a setup that links risk-taking behavior on the asset side with run decisions in a global games framework.

[8]Other work on an endogenous precision of private or public information in a global coordination game of regime change includes Hellwig and Veldkamp (2009); Szkup and Trevino (2015); Ahnert and Kakhbod (2017); Ahnert and Bertsch (2019).

[9]For a recent application of the Salop model to the competition between single-market and multiple-market banks, see Park and Pennacchi (2009).

drawn at date 1 but publicly observed at date 2. Its uniform common prior is

$$R \sim \mathcal{U}\left[R_0 - \frac{\alpha}{2}, R_0 + \frac{\alpha}{2}\right], \tag{1}$$

where $\alpha > 0$ measures investment risk and $R_0 > \frac{\alpha}{2}$ measures the expected return.

At date 0, a unit mass of atomistic investors with a unit endowment each are symmetrically located on a unit-sized circle (Figure 1). Investors have a transportation cost $\mu > 0$ per unit of distance to a bank, are indifferent between consumption at date 1 and 2, and cannot directly invest in the risky technology. N banks are equidistantly located on the circle and compete for debt funding from investors. Bank $j = 1, ..., N$ chooses opacity δ^j (more below) and offers a face value of debt D^j that result in an expected return to investors ρ^j.[10] Debt is demandable and can be withdrawn at either date 1 or 2 with face value independent of the withdrawal date.
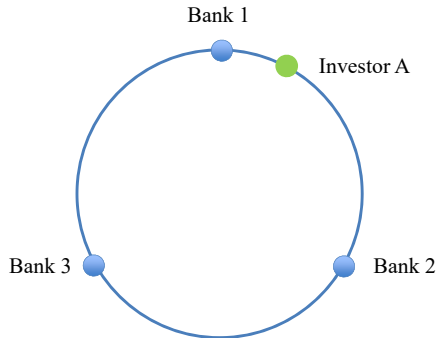

Figure 1: Location of banks on the Salop circle. Investor A has a lower transport cost to bank 1 than to bank 2.

At date 0, penniless banks are entirely funded with debt, h^j, and invest all funds, $I^j = h^j$. Liquidation at date 1 yields a fraction $0 < \psi < 1$ of the realized investment return at date 2, so the per-unit liquidation cost is $z \equiv \frac{1}{\psi} - 1 > 0$. Banks are protected by limited liability, consume at date 2, and maximize expected profits.

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[10]We assume throughout that the deposit market is covered, that is $\mu \leqslant \bar{\mu}$.

Investors delegate the decision to roll over funds at date 1 to a group of bank-specific professional fund managers $i \in [0, 1]$. If a proportion of managers $w^j \in [0, 1]$ withdraws (refuses to roll over), bank j liquidates an amount $\frac{w^j h^j D^j}{\psi R}$ to serve these withdrawals. Bank j fails due to illiquidity at date 1 and is closed early if it cannot serve interim withdrawals, $w^j h^j D^j > \psi R I^j$. If liquid, the bank's residual investment value is $R I^j - \frac{w^j h^j D^j}{\psi}$ at date 2. Bank j fails due to insolvency at date 2 if it cannot serve residual withdrawals $(1 - w^j) h^j D^j$:

$$R - \frac{w^j D^j}{\psi} < (1 - w^j) D^j. \tag{2}$$

We assume that investors recover zero upon bank failure at either date for simplicity.[11]

The simultaneous rollover decisions are governed by the compensation of fund managers. If the bank fails, a manager's relative compensation from withdrawing is a benefit $b > 0$. Otherwise, the relative compensation from withdrawing is a cost $c > 0$. Let $\gamma \equiv \frac{b}{b+c} \in (0, 1)$ summarize these parameters, where greater conservativeness (higher γ) makes fund managers more reluctant to roll over debt. This specification ensures global strategic complementarity in rollover decisions (Vives, 2005, 2014).

We assume incomplete information about the investment return at date 1 to ensure a unique equilibrium. In addition to the common prior in (1), each fund manager i receives a noisy private signal about the return (Morris and Shin, 2003):

$$x_i^j = R + \epsilon_i^j, \quad \epsilon_i^j \sim \mathcal{U}\left[-\frac{\delta^j}{2}, \frac{\delta^j}{2}\right], \tag{3}$$

where the idiosyncratic noise ϵ_i^j is independent of the investment return R and independently and identically distributed across fund managers. The idiosyncratic noise is uniformly distributed with zero mean and width $\delta^j \in [\underline{\delta}, \bar{\delta}]$ chosen by bank j at date 0, where $0 < \underline{\delta} < \bar{\delta}$ measure the minimum and maximum opacity of bank assets,

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[11]Our results are qualitatively unchanged with partial recovery.

respectively. If a bank chooses a higher level of opacity, then fund managers receive more dispersed signals. Table 1 summarizes the timeline.

$t = 0$	$t = 1$	$t = 2$
1. Banks compete for funding	1. Private signals	1. Investment matures
2. Investors deposit at a bank	2. Withdrawals	2. Banks repay or default
3. Banks invest	3. Consumption	3. Consumption

Table 1: Timeline of events.

We focus on symmetric equilibrium in pure and threshold strategies. We pin down the rollover choice of fund managers, the face value of debt, and expected bank profits. We derive the opacity choice of banks and study how competition shapes funding market outcomes and bank fragility.

2.1 Rollover of debt

Dropping the bank index j, we consider the debt rollover game between fund managers at date 1. In particular, we analyze how the opacity level δ and face value of debt D of a given bank affect the withdrawal decisions.

Proposition 1. *Bank failure. In the rollover stage at date 1, there exist unique thresholds of bank failure, $R^* \equiv (1 + z\gamma)D$, and the signal, $x^* \equiv R^* + \left(\gamma - \frac{1}{2}\right)\delta$. Fund manager i rolls over debt if and only if $x_i \geqslant x^*$ and the bank fails if and only if $R < R^*$. The withdrawal proportion for a realized investment return is*

$$
w^*(R) = \begin{cases} 1 & R \leqslant R^* - (1-\gamma)\delta \equiv \underline{R} \\ \gamma + \frac{R^*-R}{\delta} & if \quad R \in \left(\underline{R}, \widetilde{R}\right) \\ 0 & R \geqslant R^* + \gamma\delta \equiv \widetilde{R} \end{cases} \tag{4}
$$

Proof. See Appendix A. ∎

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The threshold fund manager receives the private signal $x_i = x^*$ and is indifferent between rolling over and withdrawing funding. In equilibrium, the conditional probability of bank survival of the threshold manager equals the conservatism ratio, $\Pr\{R > R^* | x_i = x^*\} = \gamma$. Moreover, the withdrawal proportion at the failure threshold also equals investor conservatism, $w(R = R^*) = \gamma$. When fund managers are more conservative, the threshold manager requires a higher conditional survival probability and fund managers are more inclined to withdraw, $\frac{dw(R)}{d\gamma} \geqslant 0$.

Opacity δ affects both the signal threshold x^* and the withdrawal proportion $w^*(R)$. Fund managers are less responsive to realized returns for a more opaque bank (Figure 2).[12] Hence, there are more partial runs when the bank survives, $R \geqslant R^*$, that, as we shall see, reduces expected profits. Similarly, there are fewer partial runs when the bank fails, $R < R^*$, though banks' expected profits are always zero by limited liability. Importantly, the uniform distribution simplifies the analysis because it implies that bank opacity does not directly affect bank failure, $\frac{\partial R^*}{\partial \delta} = 0$.
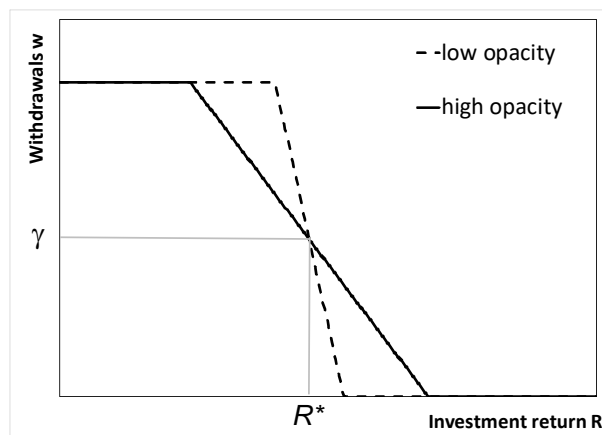

Figure 2: The impact of opacity on withdrawals: greater opacity reduces the sensitivity of the withdrawal proportion to the realized investment return.

A higher face value of debt increases the failure threshold, $\frac{dR^*}{dD} > 0$, because the

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[12]Unless stated otherwise, we use the parameters $R_0 = 1.5$, $\alpha = 0.5$, $\psi = 0.7$; $\mu = 2$, $\underline{\delta} = 0.01$, $\bar{\delta} = 0.2$, $N = 3$, and $b = c = 0.1$ so $\gamma = 0.5$ throughout.

degree of strategic complementarity among fund managers increases as the withdrawal of funds has a larger negative impact on the resources available for remaining investors (see also equation (2)). As a result, the withdrawal volume also increases, $\frac{dw^*}{dD} > 0$, for a given intermediate realization of R (see Figure 3). This effect arises indirectly via the failure threshold R^*, since the direct effect of the face value on withdrawals is zero, $\frac{\partial w^*}{\partial D} = 0$. The prior mean R_0 does not directly affect the failure threshold or the withdrawal volume but influences these indirectly via the face value of debt, as we shall see next.
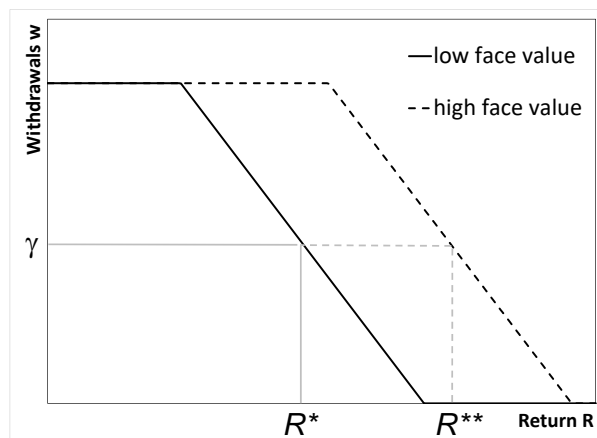

Figure 3: The impact of the face value of debt on the withdrawal volume: a higher face value increases the failure threshold and shifts out the withdrawal volume.

2.2 Funding market outcomes

We turn to the funding market at date 0. Starting with some intermediate results, we consider a bank with opacity δ and expected return to investors ρ (where we drop the bank index). We derive properties of the face value of debt D consistent with the return to investors ρ and opacity δ, and the expected per-unit bank profits π. Next, we derive the optimal levels of opacity and expected returns offered to investors.

Lemma 1. *Face value of debt.*

1. *If $\rho \leqslant \frac{R_0 - \frac{\alpha}{2}}{1 + z\gamma}$, then debt is always repaid, so $D^* = \rho$. If $\rho \geqslant \frac{(R_0 + \frac{\alpha}{2})^2}{4\alpha(1 + z\gamma)}$, then the funding market breaks down (no face value of debt exists). Otherwise, debt is risky, its face value is $D^* = \frac{R^*}{1 + z\gamma} > \rho$, and the failure threshold is*

$$R^* = \frac{y_0}{2} - \sqrt{\frac{y_0^2}{4} - \theta_1(\rho)}, \tag{5}$$

where the highest investment return is $y_0 \equiv R_0 + \frac{\alpha}{2}$ and $\theta_1(\rho) \equiv \alpha(1 + z\gamma)\rho$.

2. *When debt is risky, the failure threshold decreases and is convex in the expected investment return, $\frac{dR^*}{dR_0} < 0$ and $\frac{d^2 R^*}{dR_0^2} > 0$. The failure threshold also increases and is convex in the expected return of investors, $\frac{dR^*}{d\rho} > 0$ and $\frac{d^2 R^*}{d\rho^2} > 0$.*

Proof. See Appendix B. ∎

A main takeaway from Lemma 1 is the link between the expected return offered to investors and bank fragility. That is, the failure threshold evaluated at competitive debt pricing, R^*, increases in the expected return to investors ρ, so the range of realized investment returns for which the bank survives, $\left[R^*, R_0 + \frac{\alpha}{2}\right]$, shrinks. Hence, a higher expected return offered to investors leads to a higher bank failure probability.

There are three cases for debt pricing in Lemma 1. For a high expected investment return R_0 (relative to the return to investors ρ), debt is always repaid, so $D^* = \rho$. For a low expected investment return, the funding market breaks down because investors cannot receive the required ρ even with zero expected bank profits. For an intermediate expected investment return, debt is risky (and thus sometimes defaulted upon) but there exist a face value consistent with the return of investors. In particular, $\rho \equiv D^* \Pr\{R \geqslant R^*\}$, where the prior is used. The pricing equation has two roots and we pick the smaller one, consistent with lower bank fragility and higher expected profits. Intuitively, the failure threshold (and thus the probability of bank

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failure) fall for a higher expected investment return and a lower expected return to investors. Henceforth, we focus on parameters such that debt is risky.

Interestingly, the uniform distribution implies $D^*(\rho, \delta) = D^*(\rho)$, so the face value of debt is independent of bank opacity (once the effect of the expected returns to investors is accounted for). For ease of exposition, and without loss of generality, we state from now on the problems of banks in terms of ρ instead of D.

We turn to the expected per-unit profits. For low investment returns, $R < R^*$, the bank fails with zero profits by limited liability. Otherwise, unit profits are the return net of withdrawal costs. For intermediate returns, $R^* \leqslant R < \tilde{R}$, some withdrawals occur at date 1 even if the banker is solvent due to incomplete information and bank opacity. Some fund managers receive a low signal and withdraw—a partial run. For high returns, $\tilde{R} \leqslant R$, such withdrawals are zero. A lower bound on investment risk, $\alpha > \underline{\alpha}$, ensures that no withdrawals occur at the highest investment return, $\tilde{R} \leqslant R_0 + \frac{\alpha}{2} = y_0$, which we assume henceforth. Since the withdrawals $w(R)$ at date 1 cost $\frac{w(R)}{\psi}D$ due to partial liquidation, the bank equity E for a realized return R is:

$$E(R) \equiv \max\left\{0, R - D\big(1 + zw(R)\big)\right\}, \tag{6}$$

which is zero at the failure threshold, $E(R^*) \equiv 0$ (see also Figure 4). Integrating over all investment returns, $\pi \equiv \int_{R^*}^{y_0} E(R)\frac{1}{\alpha}dR$, yields the expected per-unit profit.

Lemma 2. *Expected per-unit profit.*

1. *For $R_0 + \frac{\alpha}{2} \geqslant 2\gamma\bar{\delta}$, the expected per-unit bank profit is*

$$\pi = \pi(R_0, \rho, \delta) \equiv -\rho + \frac{\frac{y_0^2}{2} + \theta_1(\rho) + y_0\sqrt{\frac{y_0^2}{4} - \theta_1(\rho)}}{2\alpha} - \frac{\gamma^2 z D^*}{2\alpha}\delta. \tag{7}$$

2. *The expected per-unit profit decreases and is concave in the return to investors, $\frac{d\pi}{d\rho} < 0$ and $\frac{d^2\pi}{d\rho^2} < 0$, and increases in the expected investment return, $\frac{d\pi}{dR_0} >$*
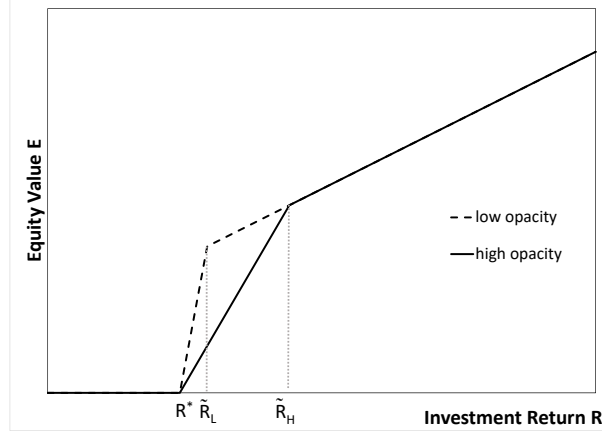
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Figure 4: The equity value of a bank at date 2 and the realized investment return. At the failure threshold, the equity value is zero, $E(R^*) = 0$. Higher bank opacity δ leads to larger partial runs and lower equity values for intermediate investment returns, $[R^*, \widetilde{R}]$, and to a higher bound $\widetilde{R}$ above which no partial runs occur ($\widetilde{R}_H > \widetilde{R}_L$).

0. *Moreover, the expected per-unit profit decreases in opacity, $\frac{d\pi}{d\delta} < 0$, with a negative cross-derivative, $\frac{d^2\pi}{d\rho d\delta} < 0$.*

Proof. See Appendix B. ∎

Higher bank opacity leads to a larger range of partial runs, $[R^*, \widetilde{R}]$, which triggers costly partial liquidation for higher investment returns. Hence, the expected per-unit profits decrease in opacity. Figure 5 shows the expected per-unit profits and its dependence on opacity, the investment return, and the return offered to investors.

Equipped with the per-unit expected profits, we turn to the equilibrium in the standard Salop model of imperfect competition for funding at date 0. The bank chooses opacity and the expected return to investors (the deposit rate) to maximize expected profits, taken as given the choices of the $N-1$ competing banks (δ^{-j}, ρ^{-j}):

$$\max_{\delta^j, \rho^j} \Pi^j = h^j(\delta^j, \rho^j)\, \pi(\delta^j, \rho^j). \tag{8}$$

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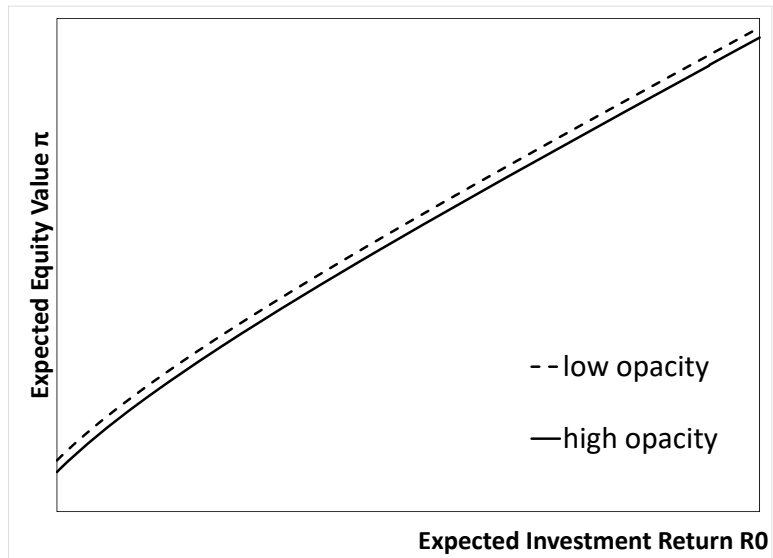
(a) expected investment return R_0
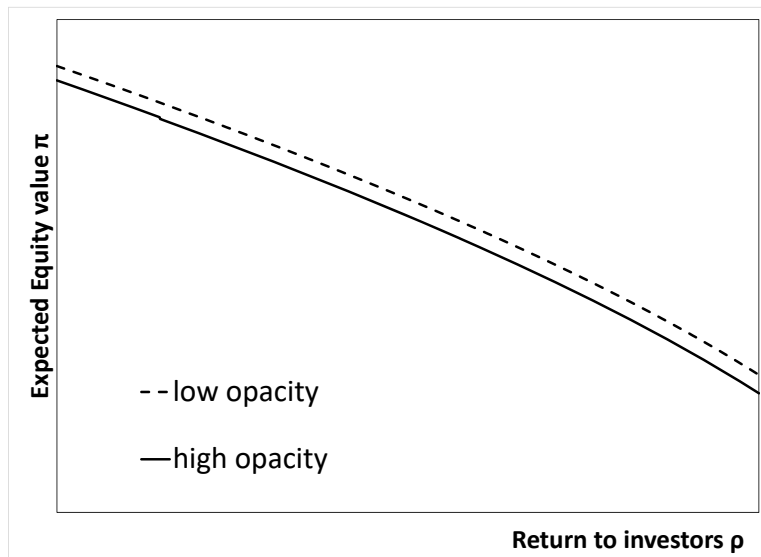
(b) return to investors ρ

Figure 5: Expected per-unit bank profits π as a function of the expected investment return R_0 (panel a), the return to investors ρ (panel b), and bank opacity δ (both panels). Higher opacity always reduces expected per-unit profits.

Proposition 2. *Opacity choice.* *Banks are as transparent as possible, $\delta^* = \underline{\delta}$.*

Proof. See Appendix C. ∎

In the static model, there is only a cost of opacity in terms of partial runs on the bank and costly interim liquidation of investment. As a result, expected per-unit profits are lower for higher opacity levels, $\frac{d\pi}{d\delta} < 0$ (see Lemma 2). Hence, banks choose to be as transparent as possible. We turn to the expected return offered to investors.

Proposition 3. *Expected returns to investors.* *The expected returns offered to investors, $\rho^* = \rho^*(R_0, N)$, is unique and implicitly given by*

$$\left.\frac{\pi}{\mu}\right|_{(\delta^*=\underline{\delta}, \rho=\rho^*)} + \frac{1}{N}\left.\frac{d\pi}{d\rho}\right|_{(\delta^*=\underline{\delta}, \rho=\rho^*)} = 0. \tag{9}$$

Greater competition increases expected returns, $\frac{d\rho^*}{dN} > 0$, and thus fragility, $\frac{dR^*}{dN} > 0$. Expected profits are $\Pi^*(R_0, N) \equiv \frac{\pi(R_0, \rho^*, \underline{\delta})}{N}$ and decrease in competition, $\frac{d\Pi^*}{dN} < 0$.[13]

Proof. See Appendix C. ∎

When offering an expected return to investors, a bank trades off its volume of funding with the expected profits per unit of funding. In the symmetric equilibrium, the each bank attracts funding of $h^* = \frac{1}{N}$. A larger number of banks induces banks to compete more fiercely for funding and, in equilibrium, results in higher expected returns to investors and lower expected bank profits. Higher expected returns to investors result in a higher face value of debt (Lemma 1) that, in turn, lead to higher bank fragility (Proposition 1). Figure 6 visualizes the impact of bank competition on funding market outcomes and bank fragility.

Proposition 4 states how bank fragility responds to a reduction in the lower

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[13]If lower transportation costs are used as a measure of greater competition between banks, we similarly get higher expected returns, $\frac{d\rho^*}{d\mu} < 0$, and thus higher fragility, $\frac{dR^*}{d\mu} < 0$.
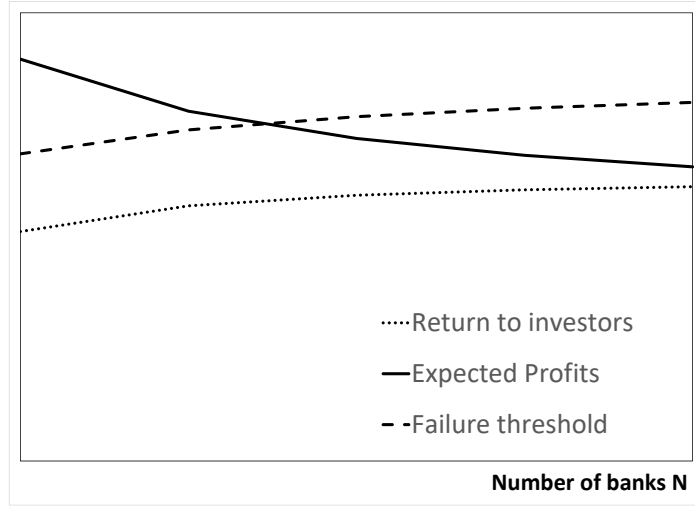
Figure 6: Expected return to investors, ρ^*, expected bank profits, Π^*, and the equilibrium failure threshold, $R^*(\rho^*)$, as bank competition N varies.

bound of opacity, $\underline{\delta}$. This change can be linked to secular changes in technology.[14]

Proposition 4. *Greater transparency.* *Lower minimum opacity increases the expected return offered to investors, $\frac{d\rho^*}{d\underline{\delta}} < 0$, and therefore increases fragility, $\frac{dR^*}{d\underline{\delta}} < 0$.*

Proof. See Appendix C. ∎

A reduction in minimum opacity increases both expected per-unit profits, $\pi_\delta < 0$ from Lemma 2, and its marginal change with respect to expected returns to investors, $\pi_{\rho\delta} < 0$. As a result, competition in funding market is fiercer and expected returns to investors are higher. In turn, banks offer a higher face value of debt that raises bank fragility (Proposition 1). Formally, this result arises from

$$\frac{dR_2^*}{d\underline{\delta}} = \frac{\partial R_2^*}{\partial \underline{\delta}} + \frac{\partial R_2^*}{\partial \rho_2^*}\frac{\partial \rho_2^*}{\partial \underline{\delta}} > 0, \tag{10}$$

where our choice of a uniform distribution highlights the role of bank competition.

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[14]We study policies and regulation mandating higher transparency (e.g., pillar 3 of the Basel bank regulation, the IFRS accounting standard, or bank stress tests) in the dynamic model in section 3.

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This distribution implies no direct effect of bank opacity on the failure threshold, $\frac{\partial R^*}{\partial \underline{\delta}} = 0$, and thus allows us to cleanly identify the indirect effect of competition.

At the heart of this result is the impact on the expected per-unit profits and its implications for expected returns to investors and bank fragility. To expand on this point, we consider an extension with an exogenous reduction in bank profits and study its effect on bank fragility. We assume a non-pecuniary per-unit cost of lending, $\ell > 0$ (such as variable operational costs, for example). Proposition 5 summarizes.

Proposition 5. ***Bank profits and bank fragility.*** *With a non-pecuniary per-unit cost of lending, ℓ, bank profits and the expected return to investors are lower, $\rho_\ell^* < \rho^*$, which reduces bank fragility, $R_\ell^* < R^*$.*

Proof. See Appendix C. ∎

A higher lending cost reduces the incentives to compete for funding, so less fierce competition reduces the expected returns to investors. In turn, this reduces the face value of debt and the failure threshold, resulting in a more stable banking system. This result sharply contrasts with results obtained in a model of risk-taking on the asset side via a moral hazard problem (for example, Boyd and Nicolo (2005)). The opposite result arises in such environments because lower expected profits increase the incentives for lower effort or higher risk-taking and, thus, increase bank fragility.

3   Dynamic model

We have studied the competition intensity in a static model so far. In this section, we add entry as another form of competition. In this dynamic version of our model, a potential entrant learns from the signals of the incumbent banks. This gives banks incentives to be opaque to deter entrance and face lower future competitive intensity.

There are two periods $T = 1, 2$, each of which resembles the static model presented in section 2 (apart from the entry decision described below). The investment return R_T follows $R_T = R_{T-1} + \eta_T$, where $R_0 > \alpha$ is known, η_T is independently and identically uniformly distributed, $\eta_T \sim U[-\frac{\alpha}{2}, \frac{\alpha}{2}]$, and independent of R_T. Since R_{T-1} is publicly observed at date 0 of period T, the common prior of R_T is

$$R_T | R_{T-1} \sim U \left[R_{T-1} - \frac{\alpha}{2}, R_{T-1} + \frac{\alpha}{2} \right]. \tag{11}$$

In each period, bank j chooses its opacity level δ_T^j and face value of debt D_T^j that results in an expected return to investors ρ_T^j. The balance sheet at date 0 is $I_T^j = h_T^j$. Banks maximize the sum of expected profits in both periods, where an entrant E may be active in period 2 only. We assume that if a bank fails, it is replaced by another bank such that the degree of competition (apart from entry) is constant across periods.[15]

At date 1 of period 1, and simultaneous to the rollover decisions, the entrant E is randomly located on the circle and receives a private signal about the investment return of the nearest bank, indexed by J. Paralleling the signals received by fund managers, the entrant's signal also depends on bank opacity choices:

$$x_E = R_1 + \epsilon_E^J, \quad \epsilon_E^J \sim \mathcal{U} \left[-\frac{\delta_1^J}{2}, \frac{\delta_1^J}{2} \right], \tag{12}$$

where ϵ_E^J is independent of R_1. In sum, an incumbent bank's opacity choice affects the precision of private information of both fund managers and the entrant (if the incumbent bank is nearest to the entrant, which occurs with probability $\frac{1}{N}$).[16]

Using the private signal x_E, the entrant decides whether to pay a fixed cost $F > 0$ in order to enter and compete in period 2. The fact that the fixed cost is paid

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[15] This assumption may reflect the case of a failed bank sold in forced merger to an outside bank.

[16] We consider a symmetric information structure for private signals. Our results qualitatively generalize to an entrant's signal of the form $x_E = R_1 + \chi \epsilon_E^J$ for $0 < \chi < \infty$.

before the realized investment return is observed at date 2 captures various costly decisions banks have to make before they can operate in a given market (for instance, the creation of relevant capacities by hiring of specialized human capital, building of offices, etc.) Let N_T be the number of banks in period T. If E enters, the number of banks is $N_2 = N + 1$ in date 2, else it is $N_2 = N$.

We study perfect Bayesian equilibrium and focus on symmetric equilibrium in pure strategies and threshold strategies. Adding to the previous analysis, we pin down both the entry decision of the entrant and the opacity choice of incumbent banks.

Generalizing previous results, the failure threshold of the investment return is $R_T^* \equiv (1 + z\gamma)D_T$ and the signal threshold is $x_T^* \equiv R_T^* + \left(\gamma - \frac{1}{2}\right)\delta_T$. The withdrawal proportion is $w_T^* = w^*(R_T)$ and the highest investment return is $y_T \equiv R_T + \frac{\alpha}{2}$. We turn to a result required for the dynamic analysis and stated in Lemma 3.

Lemma 3. *Concave expected per-unit profits in period* 2. *For $\rho_2 > \frac{3y_1^2}{16\alpha(1+z\gamma)}$, the expected per-unit profits are concave in the expected investment return, $\frac{d^2\pi_2}{dR_1^2} < 0$.*

Proof. See Appendix B. ∎

A higher prior mean has three effects on the expected per-unit profits. First, it affects the failure threshold and thus the lower bound of integration but this effect is void as $E(R_T^*) = 0$. Second, it reduces partial runs in the interval $[R_2^*, \widetilde{R}_2]$, as debt is priced more cheaply and fewer withdrawals occur for a given realized investment return. Third, the upper bound of integration increases. The stated condition suffices for concave expected profits and we focus on such parameters henceforth.

3.1 Entry choice

The potential entrant E receives a signal x_E about the current investment return R_1 at date 1. This signal is informative for the entrant since investment returns are

linked over time, as specified in equation (11). Thus, the entrant forms a posterior, $f(R_1|x_E, \delta_1^J)$, and decides whether to pay the fixed cost F. Since banks are protected by limited liability, an entrant would always choose to be active in period 2 upon incurring the fixed cost F in period 1. Thus, the entrant assigns expected profits $\Pi_2^*(R_1, N+1)$ to each of the possible investment returns R_1. The expected profits in period 2 conditional on entry and given the private signal and the nearest incumbent's choice of opacity in period 1 is

$$V(x_E, \delta_1^J) \equiv \int \Pi_2(R_1, N+1) f(R_1|x_E, \delta_1^J) \, dR_1. \tag{13}$$

Proposition 6. *Entry choice and opacity.* *If $\underline{F} \leqslant F \leqslant \overline{F}$, then there is a unique interior threshold signal x_E^* such that E enters whenever $x_E \geqslant x_E^*(\delta_1^J)$. This threshold is implicitly defined by $V(x_E^*, \delta_1^J) \equiv F$ and increases in opacity, $\frac{dx_E^*}{d\delta_1^J} > 0$. The probability of deterrence, $p^* \equiv \Pr\{x_E \leqslant x_E^*\}$, also increases in opacity, $\frac{dp^*}{d\delta_1^J} > 0$.*

Proof. See Appendix D. ∎

An incumbent bank can deter the entrant by increasing opacity in period 1. Since the entrant's future expected profits are concave in the current investment return R_1 (Lemma 3), the value of entry decreases when the information about this return is more dispersed, $\frac{dV}{d\delta_1^J} < 0$, thus reducing the incentives to enter. Moreover, a higher private signal induces the entrant to form a more positive belief about the future expected investment return, $\frac{dV}{dx_E} > 0$, and thus increases the incentives to enter. The bounds on the cost of entry ensure the existence of an interior threshold, whereby entry always occurs after the best possible signal and never occurs after the worst possible signal. Hence, we analyze situations in which the entrant's decision always depends on the signal about the realized value of the current investment return.

Higher opacity has two effects on the probability of deterrence, $p^*(R_1, \delta_1^J) =$

$\Pr\{\epsilon_E^J \leqslant x_E^* - R_1 | \delta_1^J\}$. First, higher opacity increases the entry threshold, x_E^*, and thus increases the deterrence probability. Second, higher opacity widens the support of the uniformly distributed signal and the sign of this effect depends on the realized investment return R_1. Since the lowest possible realization an incumbent bank considers is R_1^* (the incumbent fails for $R_1 < R_1^*$ and has zero charter value), $x_E^* \leqslant R_1^*$ is a sufficient condition for higher opacity to increase the probability of deterrence, which is ensured by the imposed upper bound on the fixed cost. We maintain these bounds on the fixed cost of entry henceforth.

3.2 Opacity choice

Consider an incumbent bank's choices of opacity and expected return to investors in period 1. An incumbent bank maximizes the sum of expected profits in both periods and, therefore, takes into account the impact of its choices on its charter value

$$CV(R_1, \delta_1^J) \equiv p(R_1, \delta_1^J) \Pi_2^*(R_1, N) + (1 - p(R_1, \delta_1^J)) \Pi_2^*(R_1, N+1), \qquad (14)$$

which depends on the expected future investment return, R_1, and the opacity choice of the nearest incumbent, δ_1^J. An incumbent bank receives the charter value as long as it is solvent, $R_1 \geqslant R_1^*$. Thus, we can write bank j's problem at date 0 as

$$\max_{\delta_1^j, \rho_1^j} \Pi^j = \Pi_1^j + \Pi_2^j = h_1^j(\rho_1^j)\pi_1(\delta_1^j, \rho_1^j) + \int_{R_1^*}^{R_0 + \frac{\alpha}{2}} CV(R_1, \delta_1^J) \frac{1}{\alpha} dR_1, \qquad (15)$$

where the incumbent uses the prior $R_1 | R_0$ given in equation (11).

Since the entrant randomly appears on the circle, the probability of being closest to a given incumbent, $\delta_1^J = \delta_1^j$ is $\frac{1}{N}$. With probability $1 - \frac{1}{N}$, by contrast, an incumbent's opacity choice does not affect the entrant, $\delta_1^J \neq \delta_1^j$.

Proposition 7. *Opacity and funding market equilibrium in period* 1. *For*

$N > \underline{N}$, there exists a unique interior opacity choice, $\delta_1^j = \delta_1^* \in (\underline{\delta}, \bar{\delta})$, and expected return to investors, $\rho_1^j = \rho_1^*$, that are implicitly given by

$$\frac{\pi_1}{\mu}\bigg|_{(\delta_1 = \delta_1^*, \rho_1 = \rho_1^*)} + \frac{1}{N}\frac{d\pi_1}{d\rho_1}\bigg|_{(\delta_1^*, \rho_1^*)} - \frac{CV(R_1^*, \delta_1)}{\alpha}\frac{dR_1^*}{d\rho_1}\bigg|_{(\delta_1^*, \rho_1^*)} = 0, \qquad (16)$$

$$\frac{d\pi_1}{d\delta_1}\bigg|_{(\delta_1^*, \rho_1^*)} + \int_{R_1^*}^{R_0 + \frac{\alpha}{2}} [\Pi_2^*(R_1, N) - \Pi_2^*(R_1, N+1)]\frac{dp}{d\delta_1}\frac{1}{\alpha}dR_1\bigg|_{(\delta_1^*, \rho_1^*)} = 0. \quad (17)$$

Proof. See Appendix E. ∎

Compared to the static problem (or the problem in period 2), incumbent banks in period 1 internalize the impact of their the funding market choices on the future expected charter value, where a higher probability of failure decreases the charter value. When a bank marginally increases the expected return to investors, it becomes more fragile and thus increases the probability of loosing its charter value. Interestingly, it only looses the charter value at the margin, $CV(R_1^*, \delta_1^*)$, that is when the current investment return is low, and the bank keeps the charter value for high realizations of return. This result arises because the realized investment return in period 1 jointly determines (i) the failure of incumbent banks in period 1, and (ii) the expected investment return and thus the expected charter value in period 2.

In period 1, banks offer lower rates to investors compared to period 2 for two reasons. First, the future expected charter value offers incentives to be solvent more often—the third term in equation (16)—thus competing less fiercely for funding in period 1. Second, a higher opacity level (described below) reduces the expected return to investors, just as in the static problem (Proposition 4).

In contrast to the static problem, incumbent banks choose a higher opacity level in period 1 (above $\underline{\delta}$). While opacity still reduces expected profits via costly partial runs for a larger range of realized investment returns, there is also a benefit in period 1: the deterrence of entry. Greater opacity reduces the probability of entry—the

second term in equation (17). Thus, the future expected charter value is higher and incumbent banks compete less fiercely in the funding market in period 1. The lower bound on the number of incumbent banks, $\underline{N}$, ensures an interior solution, $\delta_1^* < \bar{\delta}$.

3.3 Comparative statics

To understand the importance of competitive intensity on bank fragility and opacity, we next analyze how the equilibrium in period 1 is affected by changes in N and F.

Proposition 8. *Comparative static on bank competition.* *For $N > \underline{N}$, more competition among banks increases expected returns to investors, $\frac{d\rho_1^*}{dN} > 0$, and reduces opacity, $\frac{d\delta_1^*}{dN} < 0$. As a result, fragility is higher in period 1 and both the probability of entry and expected fragility in period 2 increase.*

Proof. See Appendix F. ∎

For a sufficiently competitive market, more competition among banks (higher N) has several effects on the expected returns to investors. First, per-unit profits decrease in the expected return to investors but this effect is smaller for higher competition, so the expected returns to investors increases. Second, the marginal charter value decreases in more competition, since the expected future profits are lower, which again increases expected returns to investors. Third, more competition deters entry, $\frac{dx_E^*}{dN} > 0$ and $\frac{dp}{dN} > 0$, so this indirect and countervailing effect via the entry choice reduces the expected returns to investors. For a high degree of competition, $N > \underline{N}$, the third effect is smaller than the second effect, so the overall effect is an increase in the expected returns to investors.

More competition among banks in the funding market has several effects on bank opacity. First, the marginal cost of opacity (lower per-unit profits due to partial runs) is unaffected by banking competition. Second, the marginal charter value again

decreases in competition, which reduces opacity. Third, more competition deters entry, $\frac{dx_E^*}{dN} > 0$, so the probability of deterrence increases less in opacity, $\frac{d^2 p}{d\delta_1 dN} < 0$, which also reduces opacity. Under the sufficient condition $N > \underline{N}$, an indirect effect via the optimal choice of expected returns to investors is again small, so the overall result is that higher competition reduces opacity.

Finally, the implications of a higher deposit rate in period 1 again translate in higher current fragility because of Lemma 1. The impact of a larger number of banks, our measure of competition, on fragility in period 2 is even stronger because future competition for deposits is fiercer (higher expected return to investors in period 2) and the probability of entry also increases because of lower opacity in period 1.

Proposition 9. ***Comparative static of entry cost.*** *For $N > \underline{N}$, a lower cost of entry increases expected returns to investors and reduces opacity, $\frac{d\rho_1^*}{dF} < 0$ and $\frac{d\delta_1^*}{dF} > 0$.*

Proof. See Appendix F. ■

A lower cost of entry has two competing effects on how the probability of deterrence changes with opacity, $\frac{dp}{d\delta_1}$. First, it reduces the signal threshold below which the entrant chooses to enter, $\frac{dx_E^*}{dF} > 0$, which reduces the change in the probability of deterrence with respect to opacity. Second, a lower cost of entry also increases how the signal threshold changes with opacity, $\frac{d^2 x_E^*}{d\delta_1 dF} > 0$. For sufficiently high competition, $N > \underline{N}$, the second effect dominates and a lower cost of entry reduces the impact of greater opacity on the probability of deterrence, $\frac{d^2 p}{d\delta_1 dF} < 0$, reducing opacity.

The dynamic model implies that a higher degree of competition (higher number of incumbent banks N or lower fixed cost of entry F) reduce bank opacity δ_1^*. Evidence consistent with this testable implication is provided by Jiang et al. (2016) who show that lower regulatory barriers to competition reduce two measures of bank opacity.[17]

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[17]These measures are discretionary loan loss provisions and the frequency with which banks restate their earnings with the SEC.

3.4   Transparency regulation

The dynamic setup allows us to study the short-term and long-term effects of policies aimed at changing the level of opacity in the banking system. In particular, we consider a policy of minimum transparency, $\delta_1^j \leqslant \hat{\delta}$. Such a policy may reflect regulation mandating higher transparency (e.g., pillar 3 of the Basel bank regulation, the IFRS accounting standard, or bank stress tests). The constraint on the upper bound of opacity is non-binding in period 2, where banks choose $\delta_2^* = \underline{\delta}$. We study the case of a binding constraint in period 1, $\hat{\delta} < \delta_1^*$, where δ_1^* is the unconstrained level of opacity and $\hat{\delta}^*$ is the constrained equilibrium characterized in Proposition 10.

Proposition 10. *Minimum transparency and fragility.* *A minimum transparency policy, $\delta_T^j \leqslant \hat{\delta}$, increases expected investor returns and fragility in period 1, $\hat{R}_1^* > R_1^*$, and increases the probability of entry and expected fragility in period 2.*

Proof. See Appendix G. ∎

In period 1, implementing minimum transparency increases bank profits and results in fiercer competition in the funding market, higher expected returns to investors, and higher fragility (see also Proposition 4). Moreover, a minimum transparency policy increases the probability of entry, resulting in fiercer competition and higher expected returns to investors in period 2 in expectation (when integrated over all possible realizations of R_1). As a result, expected bank fragility in period 2 is also higher. In sum, a minimum transparency policy raises fragility in both periods.

4   Welfare

So far we have studied the impact of competition on bank fragility. Here we consider the impact of the number of banks on utilitarian welfare. Next to the expected

profits of banks and the deposit rates received by investors, welfare also comprises the total transportation costs.[18] A constrained planner takes the incomplete information structure and the privately optimal choices of opacity and deposit rates as given but can regulate the number of banks in the economy via competition policy, which we analyze in this section. To simplify the analysis, we consider the static model with $\underline{\delta} \to 0$. Hence, $\delta^* \to 0$ and $\rho^*(\underline{\delta}, N)$ are chosen by banks (see Proposition 3).

Total transportation costs TC depend negatively on the number of banks. With N banks in the economy, investors with distance within $\left[0, \frac{1}{2N}\right]$ on either side of the Nth bank's position deposit with this bank. Using d_k as measure of distance to the closest bank, the total transportation costs are

$$TC(N) = \mu \cdot 2N \int_0^{\frac{1}{2N}} d_k dd_k = \frac{\mu}{4N}. \tag{18}$$

A unit mass of deposits is raised by banks and the deposit rates are transfers between banks and its investors, so welfare is $W \equiv \pi(\delta^*, \rho^*(N)) + \rho^*(N) - TC(N)$. This expression shows the tension associated with a larger number of banks: transportation costs are reduced but the higher deposit rate increases fragility and reduces total surplus, $\Sigma \equiv \pi(\delta^*, \rho^*(N)) + \rho^*(N)$.[19] We have the following result.

Proposition 11. *Competition policy.* *A constraint planner chooses a unique number of banks, $N^* > 0$, to balance the marginal benefit of lower total transportation costs with the marginal cost of greater fragility and, hence, a larger range of failure.*

Proof. See Appendix H. ∎

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[18]For a welfare analysis of the Rochet-Vives model, we follow the approach in Ahnert et al. (2019) and mute the impact of fund managers' payoffs on welfare. That is, we set $b \to 0$ and $c \to 0$ at a rate that preserves the positive implications of this approach, where $\frac{b}{b+c} \to \gamma$ remains constant.

[19]A similar trade-off would occur in a Cournot model of imperfect competition, where the main trade-off would be between bank fragility and loan quantity (instead of transport costs).

5 Conclusion

This paper proposes a parsimonious model of imperfect competition and opacity choice of banks subject to rollover risk and examines the implications for bank failure and the effectiveness of regulatory measures. Building on Rochet and Vives (2004) and Vives (2014), we introduce imperfect competition in the funding market and model bank opacity as the precision of private information about its asset return. This signal is more precise for more transparent banks. We first offer a static model with a fixed number of banks and Salop competition and then a dynamic model with the added feature of potential entry. The model characterizes the rollover decision of fund managers to which investors delegate the withdrawal decisions and the face value of debt offered by banks. Fund managers and the entrant receive private signals.

Higher competitive intensity among banks results in a higher face value of debt that increases the strategic complementarity in fund manager withdrawal decisions. Hence, higher competition increases expected withdrawals and expected bank failure. This result is consistent with the competition-fragility view of banking. Next, greater transparency increases bank profits because it reduces the probability of withdrawals when bank asset returns are high, so banks have an incentive to be transparent. However, when bank entry is taken into account, more transparency increases the incentives of market entry (as the entrant's signal becomes more precise), lowering future profits due to intensified competition. Therefore, the dynamic model offers a theory of bank opacity that balances its dynamic benefits (lower future competition) with its static costs (higher withdrawals and lower bank profits).

We also examine the effects of regulatory policies that aim to (i) increase the competition among banks and (ii) increase the transparency of banks. Policies that increase bank competition lead to higher face values of debt and, therefore, to an increase in bank failure. Policies that increase the transparency of banks result in higher bank competition (through higher entry) but can have a destabilizing effect

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on banks. This destabilizing effect happens for two reasons: (i) a "static reason" as higher transparency leads to higher short run profits that leads banks to increase the face value of their deposits as they want to be more competitive and (ii) a "dynamic reason", whereby higher transparency increases competition that lowers the charter value of the bank. The latter reason leads banks to increase their current (and future) face values of their debt, resulting in higher current and future bank fragility. A constrained planner chooses an intermediate level of bank competition (e.g., via competition policy) that balances these costs with the benefit of reduced transportation costs of investors.

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A   Proof of Proposition 1

Figure 7 illustrates the dominance regions if the investment return R were common knowledge. If no funding is withdrawn, $w = 0$, the bank fails when the return is smaller than the face value of debt, $\check{R} \equiv D$. It is a dominant strategy for managers not to roll over funding whenever $R < \check{R}$. Likewise, if all funding is withdrawn, $w = 1$, the bank does not fail when the return exceeds $\hat{R} \equiv \frac{D}{\psi} > \check{R}$. It is a dominant strategy to roll over funding whenever $R > \hat{R}$.
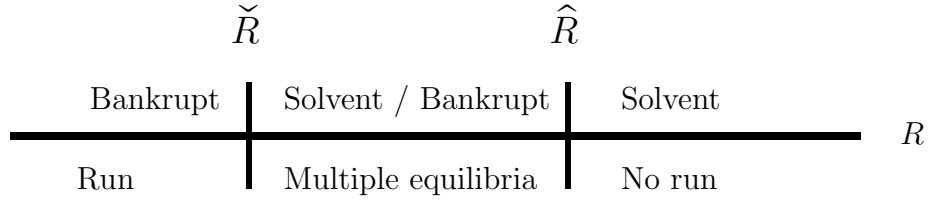
$$\check{R} \qquad\qquad\qquad \hat{R}$$

Bankrupt	Solvent / Bankrupt	Solvent
Run	Multiple equilibria	No run

R

Figure 7: Tripartite classification of investment return (complete information)

We solve for the signal and return thresholds (x^*, R^*). Since the insolvency condition is less restrictive than the illiquidity condition, the former is used (Rochet and Vives, 2004). Thus, a **critical mass condition** states that the bank fails at R^*:

$$R^* = [1 + zw(R^*)] D, \tag{19}$$

where the face value D is chosen at date 0 and the withdrawal proportion at date 1 is

$$w(R) = \Pr\{x_i < x^*|R\} = \begin{cases} 1 & R \leqslant \underset{\sim}{R} = x^* - \frac{\delta}{2} \\ \frac{x^* - R + \delta/2}{\delta} & \text{if} \quad R \in \left(\underset{\sim}{R}, \widetilde{R}\right) \\ 0 & R \geqslant \widetilde{R} = x^* + \frac{\delta}{2} \end{cases}$$

because of the distribution of ϵ_i. By Bayesian updating, the posterior distribution is $R|x_i \sim \mathcal{U}\left[x_i - \frac{\delta}{2}, x_i + \frac{\delta}{2}\right]$ for signals $R_0 - \frac{\alpha}{2} + \frac{\delta}{2} \equiv \underline{x_i} \leqslant x_i \leqslant \overline{x_i} \equiv R_0 + \frac{\alpha}{2} - \frac{\delta}{2}$. We focus on these 'non-extreme' signal here and consider and then exclude 'extreme signals' at the end of this proof. An **indifference condition** states that a fund manager receiving $x_i = x^*$ is

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indifferent between rolling over and withdrawing:

$$c \Pr\{R > R^* | x_i = x^*\} = b \Pr\{R < R^* | x_i = x^*\}. \tag{20}$$

Using the posterior distribution of $R|x^*$, the indifference condition can be expressed as $\gamma = \frac{x^* - R^* + \frac{\delta}{2}}{\delta}$. Combining both conditions yields the stated failure threshold R^* and signal threshold x^*. Inserting x^* into $\widetilde{R}$ and $\underline{R}$ yields these bounds of $w(R)$.

Finally, we consider extremely low and high signals, $x_i \leqslant \underline{x_i}$ and $x_i \geqslant \overline{x_i}$. These imply that the posterior distribution becomes triangular (non-uniform) because the boundary of the signal is close to the boundary of the possible investment return. We impose sufficient conditions to ensure that our focus on the uniform part of the posterior is appropriate. In particular, we impose a lower bound on α to guarantee that a fund manager who receives $x_i = \underline{x_i}$ strictly prefers to withdraw, and a fund manager who receives $x_i = \overline{x_i}$ strictly prefers to roll over. These conditions have to hold for any level of opacity and turn out to be most stringent for $\delta = \bar{\delta}$.

Using the posterior $R|\underline{x_i} \sim \mathcal{U}\left[R_0 - \frac{\alpha}{2}, R_0 - \frac{\alpha}{2} + \delta\right]$, a fund manager with signal $x_i = \underline{x_i}$ strictly prefers to withdraw if the conditional probability of failure strictly exceeds $1 - \gamma$, which can be expressed as $\frac{\alpha}{2} > (1 - \gamma)\bar{\delta} - R^* + R_0$. Similarly, using the posterior $R|\overline{x_i} \sim \mathcal{U}\left[R_0 + \frac{\alpha}{2} - \delta, R_0 + \frac{\alpha}{2}\right]$, a fund manager with $x_i = \overline{x_i}$ strictly prefers to roll over if the conditional probability of failure is strictly below $1 - \gamma$, which is expressed as $\frac{\alpha}{2} > \gamma\bar{\delta} + R^* - R_0$. Deriving upper and lower bounds on $R^* - R_0$ and using the bounds on the return to investors derived below, we obtain—and henceforth impose—the following sufficient lower bound:

$$\alpha \geqslant \underline{\alpha} \equiv \max\left\{4\gamma\bar{\delta}, \frac{4}{3}(1 - \gamma)\bar{\delta} + 2\frac{1 + \sqrt{2}}{3}R_0\right\}. \tag{21}$$

B   Proof of Lemmas 1, 2, and 3

We start by deriving the bounds on ρ for debt to be risky and for the funding market to break down. If debt is safe, then $D^* = \rho$ and $R^* = (1 + z\gamma)\rho$. Thus, for debt to be indeed

safe, we require $R^* \leqslant R_0 - \frac{\alpha}{2}$, which yields the stated bound on ρ.

If debt is risky instead, its pricing is $\rho = \frac{y_0 - R^*}{\alpha} D^*$. Substituting in from the failure threshold, we obtain the quadratic equation $(y_0 - R^*)R^* = \theta_1(\rho) > 0$. The maximum of the left-hand side is reached at $R^{max} = \frac{y_0}{2}$ and yields the value $\frac{y_0^2}{4}$, so the funding market exists if $y_0 \geqslant 2\sqrt{\theta_1(\rho)}$, which yields the stated bound on ρ. The smaller root of the quadratic equation is given in Lemma 1. Verifying the supposed partial default requires $R^* < R_0 + \frac{\alpha}{2} = y_0$, which always holds. The failure threshold depends on the expected investment return R_0 and the return of investors ρ as follows:

$$\frac{dR^*}{dR_0} = \frac{1}{2} - \frac{y_0}{4\sqrt{\frac{y_0^2}{4} - \theta_1}} < 0, \qquad \frac{d^2 R^*}{dR_0^2} = \frac{\theta_1}{4\sqrt{\frac{y_0^2}{4} - \theta_1}^3} > 0, \tag{22}$$

$$\frac{dR^*}{d\rho} = \frac{\alpha(1 + z\gamma)}{2\sqrt{\frac{y_0^2}{4} - \theta_1}} > 0, \qquad \frac{d^2 R^*}{d\rho^2} = \frac{\alpha^2(1 + z\gamma)^2}{4\sqrt{\frac{y_0^2}{4} - \theta_1}^3} > 0. \tag{23}$$

Expected per-unit profits are $\pi = \frac{1}{\alpha}\int_{R^*}^{y_0}\left[R - D^*(1 + zw(R))\right]dR$. We assume that there are no partial runs for a sufficiently high investment return, $\widetilde{R} < y_0$. Rewriting this yields the sufficient condition $\alpha > \underline{\alpha} \equiv \max_\delta \frac{\gamma\delta(R_0 - \gamma\delta)}{(1 + z\gamma)\rho - \frac{\gamma\delta}{2}}$, which we assume henceforth. The expected per-unit profit stated in Lemma 2 follows and changes according to:

$$\frac{d\pi}{d\rho} = -1 + \frac{1 + z\gamma}{2}\left[1 - \frac{y_0}{2\sqrt{\frac{y_0^2}{4} - \theta_1}}\right] - \frac{\gamma^2 z\delta}{2\alpha(1 + z\gamma)}\frac{dR^*}{d\rho} < 0 \tag{24}$$

$$\frac{d^2\pi}{d\rho^2} = -\frac{\alpha(1 + z\gamma)^2 y_0}{8\sqrt{\frac{y_0^2}{4} - \theta_1}^3} - \frac{\gamma^2 z\delta}{2\alpha(1 + z\gamma)}\frac{d^2 R^*}{d\rho^2} < 0 \tag{25}$$

$$\frac{d\pi}{dR_0} = \frac{1}{2\alpha}\left(y_0 + \sqrt{\frac{y_0^2}{4} - \theta_1} + \frac{y_0^2}{4\sqrt{\frac{y_0^2}{4} - \theta_1}}\right) - \frac{\gamma^2 z\delta}{2\alpha(1 + z\gamma)}\frac{dR^*}{dR_0} > 0 \tag{26}$$

$$\frac{d^2\pi}{dR_0^2} = \frac{1}{2\alpha}\left(1 + \frac{3y_0}{4\sqrt{\frac{y_0^2}{4} - \theta_1}} - \frac{y_0^3}{16\sqrt{\frac{y_0^2}{4} - \theta_1}^3}\right) - \frac{\gamma^2 z\delta}{2\alpha(1 + z\gamma)}\frac{d^2 R^*}{dR_0^2} \tag{27}$$

$$\frac{d\pi}{d\delta} = -\frac{\gamma^2 z R^*}{2\alpha(1 + z\gamma)} < 0, \qquad \frac{d^2\pi}{d\delta^2} = 0, \qquad \frac{d^2\pi}{d\delta d\rho} = -\frac{\gamma^2 z}{2\alpha(1 + z\gamma)}\frac{dR^*}{d\rho} < 0. \tag{28}$$

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Thus, the expected profits is concave in the expected investment return (see equation (27)) if $y_0^2 < \frac{16}{3}\theta_1$. Forwarding to period $T = 2$ yields the stated bound on ρ_2.

C   Proof of Propositions $2 - 5$

We turn to the Salop model of competition at date 0. The return of some investor k from depositing with bank j is $\rho^j - \mu d_k^j$. For a fully covered market, $\mu \leqslant \bar{\mu}$, we can focus on the two banks nearest to investor k, labeled as 1 and 2. Their distance is d_k and $\frac{1}{N} - d_k$ because banks are equidistant on the unit circle (Figure 1). Hence, the location at which investor k is indifferent between going to either bank is

$$d_k^* = \frac{\rho^1 - \rho^2}{2\mu} + \frac{1}{2N}. \tag{29}$$

Total funding supply comes from both sides relative to a bank's location on the circle, so the amount of funding is $2d_k^*$ or $h^j = \frac{\rho^j - \rho^{-j}}{\mu} + \frac{1}{N}$, which is independent of the opacity choice, $\frac{dh^j}{d\delta^j} = 0$, but increases in the return offered to investors, $\frac{dh^j}{d\rho^j} = \frac{1}{\mu} > 0$.

The total derivatives of (total) expected profits are:

$$\frac{d\Pi^j}{d\delta^j} = \frac{dh^j}{d\delta^j}\pi^j + h^j \frac{d\pi^j}{d\delta^j} < 0, \qquad \frac{d\Pi^j}{d\rho^j} = \frac{dh^j}{d\rho^j}\pi^j + h^j \frac{d\pi^j}{d\rho^j}. \tag{30}$$

Thus, $\delta^j = \delta^* = \underline{\delta}$ for all j. The first-order condition for the expected return evaluated at the symmetric equilibrium, $h^* = \frac{1}{N}$, is given in (9). The second-order condition is $\frac{d^2\Pi}{d\rho^2} = \frac{2}{\mu}\frac{d\pi}{d\rho} + h\frac{d^2\pi}{d\rho^2} < 0$, so we have a unique solution that is a global maximum. Since $\frac{d^2\Pi^j}{d\rho^j dN} = -\frac{d\pi^j}{d\rho^j}\frac{1}{N^2} > 0$, we obtain $\frac{d\rho^*}{dN} > 0$ from the implicit function theorem. Thus, $\frac{dR^*}{dN} > 0$ follows from $\frac{dR^*}{d\rho} > 0$ (Lemma 1). Similarly, $\frac{d^2\Pi^j}{d\rho^j d\mu}\Big|_{\rho=\rho^*} = -\frac{1}{\mu^2}\pi^j < 0$, so we obtain $\frac{d\rho^*}{d\mu} < 0$ from the implicit function theorem and $\frac{dR^*}{d\mu} < 0$ follows. Next, using the envelope theorem, we have $\frac{d\Pi^*}{dN} = -\frac{\pi(R_0, \rho^*(R_0, N), \underline{\delta})}{N^2} < 0$.

For Proposition 4, we consider changes in $\underline{\delta}$. The implicit function theorem implies $\frac{d\rho^*}{d\underline{\delta}} < 0$ because $\frac{d^2\Pi}{d\rho^2} < 0$ (see above) and $\frac{d^2\Pi}{d\rho d\delta} = \frac{1}{\mu}\frac{d\pi}{d\delta} + h\frac{d^2\pi}{d\rho d\delta} < 0$. The remaining results

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are immediate from Lemma 1 and Proposition 1.

For Proposition 5, similar results obtain in the extension with a pecuniary cost of lending ℓ. The expected profits of the bank change to $\Pi_\ell = h\left[\pi(\delta, \rho) - \ell\right]$, where we drop the bank index again. Thus, $\frac{d\Pi_\ell}{d\delta} = \frac{d\Pi}{d\delta} < 0$ and $\delta^* = \underline{\delta}$. Moreover, $\frac{d\Pi_\ell}{d\rho} = \frac{d\Pi}{d\rho} - \frac{\ell}{\mu}$, so $\frac{d^2\Pi_\ell}{d(\rho)^2} = \frac{d^2\Pi}{d(\rho)^2} < 0$ and $\frac{d^2\Pi_\ell}{d\rho d\ell} = -\frac{1}{\mu} < 0$. By the implicit function theorem, $\frac{d\rho^*}{d\ell} < 0$ and $\rho_\ell^* < \rho^*$. The result on bank fragility is immediate from Lemma 1 and Proposition 1.

D   Proof of Proposition 6

The entrant uses the signal x_E to form a posterior, $R_1|x_E \sim U\left[x_E - \frac{\delta^J}{2}, x_E + \frac{\delta^J}{2}\right]$ (where we again exclude the extreme signals, see below). Thus, expected profits are

$$V(x_E, \delta^J) \equiv \int_{x_E - \frac{\delta^J}{2}}^{x_E + \frac{\delta^J}{2}} \frac{1}{N+1} \pi_2(R_1, \rho_2^*(R_1, N+1), \delta_2^*) \frac{1}{\delta^J} dR_1, \tag{31}$$

where $\delta_2^* = \underline{\delta}$ and ρ_2^* is given by the first-order condition in equation (9), and we drop the period index on δ henceforth.

Using a threshold strategy, so the entrant enters whenever $x_E \geqslant x_E^*$. This threshold is implicitly defined by an indifference condition between entering and not entering upon receiving the threshold signal, $V(x_E^*, \delta^J) \equiv F$, for any opacity choice δ^J. Using the envelope theorem, expected profits change according to:

$$\frac{dV}{dx_E} = \frac{1}{N_2 \delta^J}\left[\pi_2\left(x_E + \frac{\delta^J}{2}, \cdot, \cdot\right) - \pi_2\left(x_E - \frac{\delta^J}{2}, \cdot, \cdot\right)\right] > 0 \tag{32}$$

$$\frac{dV}{d\delta^J} = \frac{1}{\delta^J}\left[\frac{\pi_2\left(x_E + \frac{\delta^J}{2}, \cdot, \cdot\right) + \pi_2\left(x_E - \frac{\delta^J}{2}, \cdot, \cdot\right)}{2N_2} - V(x_E, \delta^J)\right] < 0, \tag{33}$$

where the first inequality arises from $\frac{d\pi_2}{dR_1} > 0$ (Lemma 2) and implies that the solution is unique if it exists. This strong monotonicity of expected profits in the signal x_E implies that our focus on threshold threshold strategies is without loss of generality. The second

inequality follows from concavity. By the implicit function theorem, we have:

$$\frac{dx_E^*}{d\delta^J} = -\left.\frac{\frac{dV}{d\delta^J}}{\frac{dV}{dx_E}}\right|_{x_E=x_E^*} = \frac{(N+1)F - \frac{\pi^+ + \pi^-}{2}}{\pi^+ - \pi^-} > 0. \tag{34}$$

The existence of an interior solution is guaranteed by appropriate bounds on the cost of entry. To exclude the part of the posterior that is a triangular distribution, we require certain entry for the signal $x_E = \overline{x_E} \equiv R_0 + \frac{\alpha}{2} - \frac{\delta^J}{2}$ and certain non-entry for $x_E = \underline{x_E} \equiv R_0 - \frac{\alpha}{2} + \frac{\delta^J}{2}$. For such behavior to arise for any opacity level, we impose:

$$F \leqslant \tilde{F} \equiv V\left(R_0 + \frac{\alpha}{2} - \frac{\bar{\delta}}{2}, \bar{\delta}\right), \qquad F \geqslant \underline{F} \equiv V\left(R_0 - \frac{\alpha}{2} + \frac{\bar{\delta}}{2}, \underline{\delta}\right). \tag{35}$$

Thus, the probability of deterrence is $p(R_1, \delta^J) = \Pr\{R_1 + \epsilon_E^J \leqslant x_E^*\} = \frac{1}{2} + \frac{x_E^* - R_1}{\delta^J}$ and changes according to:

$$\frac{dp^*}{d\delta^J} = \frac{1}{\delta^J}\left[\frac{dx_E^*}{d\delta^J} - \frac{x_E^* - R_1}{\delta^J}\right]. \tag{36}$$

Since any incumbent bank fails for $R_1 < R_1^*$, a sufficient condition for $\frac{dp^*}{d\delta^J} > 0$ is $x_E^* \leqslant R_1^*$. A sufficient condition for this is $F \leqslant V(R_1^*, \bar{\delta}) \equiv \hat{F}$, where $\overline{F} \equiv \min\{\hat{F}, \tilde{F}\}$.

E   Proof of Proposition 7

The second-order derivatives are

$$\frac{d^2\Pi^j}{d(\rho^j)^2} = \frac{2}{\mu}\frac{d\pi_1^j}{d\rho_1^j} + h_1^j\frac{d^2\pi_1^j}{d(\rho_1^j)^2} - \frac{CV(R_1^*, \delta^J)}{\alpha}\frac{d^2R_1^*}{d(\rho_1^j)^2} - \frac{dCV(R_1^*, \delta^J)}{\alpha dR_1^*}\left(\frac{dR_1^*}{d\rho_1^j}\right)^2 < 0 \tag{37}$$

$$\frac{d^2\Pi^j}{d(\delta^j)^2} = \frac{1}{N\alpha}\int_{R_1^*}\left(\Pi_2^*(R_1, N) - \Pi_2^*(R_1, N+1)\right)\frac{d^2p}{d\delta^2}dR_1 \tag{38}$$

$$\frac{d^2\Pi^j}{d\delta^j d\rho^j} = \frac{1}{\mu}\frac{d\pi_1^j}{d\delta_1^j} + h_1^j\frac{d^2\pi_1^j}{d\rho_1^j d\delta_1^j} - \frac{1}{N\alpha}\frac{dR_1^*}{d\rho_1^j}\left.\frac{dp}{d\delta^J}\right|_{R_1^*}\left(\Pi_2^*(R_1^*, N) - \Pi_2^*(R_1^*, N+1)\right) < 0.$$

Thus, a sufficient condition for $\frac{d^2\Pi^j}{d(\delta^j)^2} < 0$ is $\frac{d^2p}{d\delta^2} < 0$, which can be expressed as

$$\frac{d^2p}{d\delta^2} = \frac{\frac{d^2x_E^*}{d\delta^2}}{\delta} - 2\frac{\frac{dx_E^*}{d\delta}}{\delta^2} + \frac{x_E^* - R_1^*}{\delta^3}. \tag{39}$$

Since the second term is negative and the third term is weakly negative, a sufficient condition is $\frac{d^2x_E^*}{d\delta^2} \leqslant 0$. By differentiation, this term is

$$\begin{aligned}
\frac{d^2x_E^*}{d\delta^2} &= \frac{dx_E^*}{d\delta}\Big[d\pi^+(\pi^- - (N+1)F) + d\pi^-((N+1)F - \pi^+)\Big] + \\
&\quad \frac{1}{2}\Big[d\pi^+(\pi^- - (N+1)F) - d\pi^-((N+1)F - \pi^+)\Big],
\end{aligned} \tag{40}$$

where we used the shorthands $\pi^+ \equiv \pi_2^*(x_E^* + \frac{\delta}{2})$, $\pi^- \equiv \pi_2^*(x_E^* - \frac{\delta}{2})$, and $d\pi \equiv \frac{d\pi_2^*}{dy_1}$. A sufficient condition is $d\pi^+(\pi^- - (N+1)F) - d\pi^-((N+1)F - \pi^+) \leqslant 0$, because the terms multiplying $\frac{dx_E^*}{d\delta}$ is negative. This condition holds by concavity. Thus, $\frac{d^2p}{d\delta^2} < 0$ and the concavity of the objective function ensures uniqueness.

For $\delta^j \to 0$, the marginal cost of opacity vanishes, while the marginal benefit is strictly positive, so $\delta^* > 0$. By continuity, there is an interior solution for small enough $\underline{\delta} > 0$. Also, $\delta^* < \bar{\delta}$ is ensured by $N > \underaccent{\sim}{N}_1$, where $\underaccent{\sim}{N}_1$ solves $0 = -\frac{\gamma^2 z R_1^*(\rho_1^*(\underaccent{\sim}{N}_1))}{2\alpha(1+z\gamma)}\bar{\delta} + \frac{1}{\alpha}\int_{R_1^*}\Big(\Pi_2^*(R_1, \underaccent{\sim}{N}_1) - \Pi_2^*(R_1, \underaccent{\sim}{N}_1 + 1)\Big)\frac{dp}{d\delta}\Big|_{\bar{\delta},\underaccent{\sim}{N}_1} dR_1$, where we abstract from integer constraints. For a definition of $\underline{N}$ and its relation to $\underaccent{\sim}{N}_1$, see Appendix F.

F   Proof of Propositions 8 and 9

From the implicit function theorem and equation (31), we have $\frac{dx_E^*}{dF} = \frac{\delta(N+1)}{\pi^+ - \pi^-} > 0$ and $\frac{dx_E^*}{dN} = \frac{F\delta}{\pi^+ - \pi^-} > 0$. Using equation (34), we obtain the partial derivatives for the entry threshold $\frac{d^2x_E^*}{d\delta dN} = \frac{F}{\pi^+ - \pi^-}$ and $\frac{d^2x_E^*}{d\delta dF} = \frac{N+1}{\pi^+ - \pi^-} > 0$. Thus, the partial derivatives for the probability of deterrence are $\frac{d^2p}{d\delta dN} = 0 = \frac{d^2p}{d\delta dF}$. Using the first-order conditions evaluated at the symmetric equilibrium, we obtain the following partial derivatives w.r.t. N and F:

$\frac{d^2\Pi}{d\delta dF} = 0$, $\frac{d^2\Pi}{d\delta dN} = \frac{1}{\alpha}\int_{R_1^*}\frac{dp}{d\delta}\left(\frac{d\Pi_2^*(N)}{dN} - \frac{d\Pi_2^*(N+1)}{dN}\right)dR_1 < 0$ and

$$
\begin{aligned}
\frac{d^2\Pi}{d\rho_1 dN} &= -\frac{1}{N^2}\frac{d\pi_1}{d\rho_1} - \frac{1}{\alpha}\frac{dR_1^*}{d\rho_1}\left[\frac{dp}{dN}(\Pi_2^*(N) - \Pi_2^*(N+1)) + p\frac{d\Pi_2^*(N)}{dN} + (1-p)\frac{d\Pi_2^*(N+1)}{dN}\right] \\
\frac{d^2\Pi}{d\rho_1 dF} &= -\frac{1}{\alpha}\frac{dR_1^*}{d\rho_1}\frac{dp}{dF}\left[\Pi_2^*(N) - \Pi_2^*(N+1)\right] < 0
\end{aligned}
\tag{41}
$$

Since $p \geqslant 0$, a sufficient condition for $\frac{d^2\Pi}{d\rho_1 dN} > 0$ is $N \geqslant \underset{\sim}{N}_2 \equiv \frac{V}{\pi^+ - \pi^-}$. Let $\underline{N} \equiv \max\{\underset{\sim}{N}_1, \underset{\sim}{N}_2\}$ and we impose $N > \underline{N}$ henceforth. Since the determinant of the Jacobian is $|J| \equiv \frac{d^2\Pi}{d\rho_1^2}\frac{d^2\Pi}{d\delta^2} - \left(\frac{d^2\Pi}{d\rho_1 d\delta}\right)^2 > 0$, the implicit function theorem with the above partial derivatives yields the stated comparative statics. For example, $\frac{d\rho_1^*}{dN} = -|J|^{-1}(\Pi_{\rho N}\Pi_{\delta\delta} - \Pi_{\delta N}\Pi_{\rho\delta}) > 0$ and $\frac{d\delta_1^*}{dF} = -|J|^{-1}(\Pi_{\rho\rho}\Pi_{\delta F} - \Pi_{\rho\delta}\Pi_{\rho F}) > 0$. Finally, the results for fragility follow from Lemma 1 and we get $\frac{dR_1^*}{dN} > 0$.

G Proof of Proposition 10

The optimality of the unregulated equilibrium implies $\delta^* = \hat{\delta}$ and $\hat{\rho}_1^* > \rho_1^*$, where hats denote the constrained equilibrium levels. This results arises from the implicit function theorem and $\frac{d^2\Pi^j}{d(\rho^j)^2} < 0$ and $\frac{d^2\Pi^j}{d\rho^j d\delta^j} < 0$. As in the static case, fragility is higher. Let the expected probability of deterrence be $P = \mathbb{E}[p(R_1, \delta)] \equiv \int_{R_0 - \frac{\alpha}{2}}^{R_0 + \frac{\alpha}{2}} p(R_1, \delta)\frac{1}{\alpha}dR_1$, so $\frac{dP}{d\delta} > 0$. Hence, $\hat{\delta} < \delta^*$ implies $\hat{P} < P^*$. The result on expected fragility in period 2 follows from $\mathbb{E}[R_2^*] \equiv P\,R_2^*(N) + (1-P)\,R_2^*(N+1)$ and the ranking of δ.

H Proof of Proposition 11

Welfare changes with the number of banks according to

$$
\frac{dW}{dN} = \frac{\mu}{N^2}\left[\frac{1}{8} + \frac{1+z\gamma}{2}\left(1 - \frac{y_0}{2\sqrt{\frac{y_0^2}{4} - \theta_1}}\right)\frac{1}{1 + \frac{\mu\pi_{\rho\rho}}{N\pi_\rho}}\right],
\tag{42}
$$

where $\pi_\rho \equiv \frac{d\pi}{d\rho}$ and $\pi_{\rho\rho} \equiv \frac{d^2\pi}{d\rho^2}$ are given in Appendix B. The first term is the marginal benefit of a larger number of banks via lower transportation costs. The second term is the marginal cost of a larger number of banks via lower surplus to banks and investors because of greater fragility. Since ρ^* increases in N and $\frac{\pi_{\rho\rho}}{\pi_\rho}$ decreases in N, the marginal cost decreases in N. Therefore, $\frac{d^2W}{dN^2} < 0$ and a unique N^* exists. Since the marginal cost at $N = 0$ is zero, while the marginal benefit is large, it follows that $N^* > 0$.